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Power Rationing in a Long-term Power Shortage

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Abstract

In many developing countries power demand is much greater than can be met, leading to routine load shedding. Some policy must be chosen for the fair and efficient rationing of power, however, transmission constraints and unreliable generation can make it difficult to achieve a particular allocation of power.

We develop methods to quantify the trade-off between maximising the total amount of power delivered and distributing the available power in a fairer way. To do so, we model a common situation in which the system operator minimises load shedding, subject to exogenous proportional allocation targets for different regions. We then explore how the level of permitted deviation from the target affects the level of load shedding. This minimisation problem is mathematically challenging, but we develop an efficient solution method for it based on Lagrangian decomposition.

We apply our methods to a case study of the Nigerian Power system and analyse the Pareto frontiers between efficiency and fairness obtained under different specifications of the problem. In our case study, we show that current Nigerian policies reduce the total amount of power delivered by up to 5%, but that enforcing policies over a longer time horizon substantially reduces this efficiency penalty.

Index Terms

Optimal Power Flow, Power Rationing, Load Shedding, Developing Countries, Nigeria, Lagrangian Decomposition

Power Rationing in a Long-term Power Shortage

1 INTRODUCTION

In many developing countries power demand is much greater than supply, leading to frequent load shedding for a large proportion or even all of the daily load cycle. Nigeria [1] and Pakistan [2] present egregious cases of chronic power crises. News reports of countries or regions in similar situations can be found from India [3], Brazil [4], South Africa [5], Zimbabwe [6] and Serbia [7] to name just a few. Power outages in developing countries have been shown to cause a substantial drag on economic growth [8] because demand is either unmet or has to be supplied by self-generation at a high cost.

In the long run additional investment to improve the electricity supply industry is of crucial importance for economic development in many countries. Nevertheless, in the short to medium term it is also important to make the best use of existing infrastructure, which means that research addressing the technical and economic issues of power system operation in conditions of electricity shortage is needed. However the literature on this topic is limited.

Sophisticated models have been proposed for efficient allocation of service interruptions in power shortage conditions in [9], [10], [11] and [12]. However, these rely on complex financial mechanisms which are unlikely to be practical in developing countries where institutional competence is often poor [13]. There is literature on optimal load shedding such as [14], [15] and [16]. However, these treat load shedding as an emergency measure to return the system to a steady state in the case of a contingency event. The contribution of our work is to consider the implications of the case when load shedding is a constant necessity for the operation of the system. [17] discusses how power rationing can be most effectively implemented to deal with long-term power shortages and the authors present case studies from developed and developing countries, including Chile, China, California, the Dominican Republic, Japan and Brazil. [18] attempts to draw lessons from the Californian experience of power rationing in 2001 that can be applied to developing countries. These are both empirical studies focusing on institutional and managerial issues. In contrast our work provides a decision support model targeted specifically at the needs of developing countries.

In this paper we develop a modeling framework to help energy policy makers quantify the trade off between maximising the proportion of energy demand served and distributing the burden of load shedding in an equitable way between customers in different regions of the network. We model this as a multi-objective optimisation problem and show how to analyse the trade off by obtaining the Pareto frontier of the two objectives. We apply this framework to a model of the Nigerian power system in 2014 and show how transmission constraints limit what distribution of power is possible on any individual day. We then show that achieving fairness over a long period allows more energy to be supplied, but at the cost of a significantly more variable level of power supply on a day-to-day basis.

The remainder of this paper is organised as follows: In Section 2 we describe the motivating situation and explain how our modeling approach captures its important features. Section 3 describes our model and the solution methods we use to obtain our results. In Section 4 we present a case study of the Nigerian Power System. This is divided into 5 subsections: Section 4.1 gives some background information on the Nigerian power system; Section 4.2 describes the network model; Section 4.3 presents results demonstrating the importance of using the AC power flow equations in this case; Section 4.4 describes the set of operating points used to approximate the range and frequency of different network operating conditions; and in Section 4.5 we present the results of our analysis.

2 MOTIVATION

When a power system is not able to meet demand, some policy must be chosen for the fair and efficient rationing of power. However, in underdeveloped power systems the transmission network may be highly constrained. Furthermore, poor maintenance of generators and unreliable supplies of fuel mean that the level and spatial distribution of available generation is highly variable. The system operator must take these into account in order to determine an optimal policy of load shedding and generation.

We analyse the case when the system operator wants to dispatch generation and ration power supply in order to achieve a desired balance between maximising power delivery and supplying a fixed proportion of total power supplied to different regions of the network. We have chosen to study this particular formulation of the problem because there is a concrete example of this type of regulation implemented in Nigeria (see Section 4), but our methods are applicable to other similar settings.

We formulate this problem as a multi objective optimisation problem. The first objective is to minimise the total load shed. The second objective is to minimise a measure of the deviation from the proportional targets in all regions that have a shortfall.

We consider two cases resulting in two problem formulations. In the first case the system operator is concerned with achieving a desired balance between proportional fairness and power delivery over a short time horizon during which the operating conditions of the power system (i.e. the level of demand and available generation) are expected to remain constant. We call this the Short-term Load Allocation Problem. In the second case the system operator wishes to achieve the desired balance between proportional fairness and power delivery on average, over a longer time period. We call this the Long-term Load Allocation Problem.

We model the variation in the operating conditions of the network by a set of operating points representing the range and relative frequency of different operating conditions. For each operating point we define a power flow problem and impose constraints defining proportional targets for power supply to each region of the network. In the Short-term Load Allocation Problem these constraints must be satisfied in each operating point so regions in shortfall with respect to the target in one operating point cannot be compensated for this by a surplus in other operating points. In the Long-Term Load Allocation Problem the proportional target constraints apply to the power supply aggregated over all operating points so that shortfall in one operating point can be compensated by a surplus in another operating point.

For this paper we neglect issues of uncertainty about the future level of available generation and instead assume that the uncertain future availability levels are represented by a set of operating points. This is a valid assumption if the system operator is concerned with a policy that is optimal, on average, over the long term, and the distribution of the level of available generation is stationary.

We can model the physics of power flows at varying levels of detail by using either the AC power flow equations or their DC linearisation. The AC version more accurately simulates the power flows in the network, including the effects of reactive power, transmission losses and line capacity limits. The advantage of the DC version is that the resulting Optimal Power Flow problem is a convex problem, meaning that it can be solved more quickly and to proven optimality. The DC power flow equations are widely used in techno-economic electricity models because they are a good approximation to the true behavior of most power systems, operating in normal conditions. However, for a power system operating close to its limits it is important to precisely model reactive power flows and transmission losses (see Section 4.3).

3 METHODOLOGY

3.1 Model

The constraints of our model are described by equations (2) to (15) and either (16) or (18) using the sets, parameters and variables in Table 1. This formulation uses the AC power flow equations for each operating point defined using in polar coordinates as in [19]. We formulate the DC version by removing all reactive power constraints and variables from the problem and assuming that the voltage magnitude, $v = 1$ for all buses.

The objective function to be minimised for the Short-term Problem is given by equation (1). This is the weighted sum of a penalty for violating the load allocation constraints in every operating point minus the sum of power supplied in all operating points. Constraint (15) defines the difference, $d_{r,t}$, between the power supplied in region r at operating point t and the corresponding target power supply for the region. The target power supply for region r is given by the total power supply to all regions multiplied by P_r , the proportion of power that should flow to region r according to the load allocation regulation. The shortfall in region r at operating point t , $s_{r,t}$, defined in constraint (16), is the positive part of $d_{r,t}$. In the context of this paper "shortfall" is the gap between the amount of power supplied in a region and the amount it should get if the proportional targets were respected. It does not mean the amount of unmet demand.

The objective function to be minimised for the Long-term Problem is given by Equation (17). This is the weighted sum of a penalty for violating the load allocation constraints considering all operating points together minus the sum of power supplied in all operating points. The long-term shortfall, s_r^L defined in constraint (18) is the positive part of the difference terms summed over all operating points t . It is defined for every region.

TABLE 1: Symbols used to define the OPF

Sets	
\mathcal{G}	Generators, indexed by g
\mathcal{B}	Buses, indexed by b
\mathcal{D}	Demands, indexed by d
\mathcal{G}_b	Generators at bus b
\mathcal{D}_b	Loads at bus b
\mathcal{D}_r	Loads in region r
\mathcal{B}_b	Buses connected to bus b
\mathcal{R}	Regions, indexed by r
\mathcal{T}	Operating points, indexed by t
b_0	slack bus
Parameters	
V_b^{UB}	Voltage magnitude upper bound at bus b
V_b^{LB}	Voltage magnitude lower bound at bus b
$P_{g,t}^{\text{UB}}$	Real power generation upper bound for generator g
$P_{g,t}^{\text{LB}}$	Real power generation lower bound for generator g
$Q_{g,t}^{\text{UB}}$	Reactive power generation upper bound for generator g
$Q_{g,t}^{\text{LB}}$	Reactive power generation lower bound for generator g
$P_{d,t}^{\text{D}}$	Real power at load d
$Q_{d,t}^{\text{D}}$	Reactive power at load d
$S_{bb'}^{\text{max}}$	Apparent power limit on line bb'
$G_{bb'}$	Conductance of line bb'
$B_{bb'}$	Susceptance of line bb'
P_r	Proportional load allocation target for region r
L^s	Limit on total load allocation slack variables
W	Weighting parameter for shortfall minimisation objective
Variables	
$v_{b,t}$	Voltage magnitude at bus b , in operating point t
$p_{g,t}^{\text{G}}$	Real power generation at generator g , in operating point t
$q_{g,t}^{\text{G}}$	Reactive power generation at generator g , in operating point t
$\alpha_{d,t}$	Proportion of load shed at load d , in operating point t
$p_{d,t}^{\text{D}}$	Real power consumed by load d in operating point t
$p_{r,t}^{\text{R}}$	Real power consumed by loads in region r in operating point t
$q_{d,t}^{\text{D}}$	Reactive power consumed by load d in operating point t
$P_{bb',t}^{\text{L}}$	Real power on line bb' from bus b , in operating point t
$Q_{bb',t}^{\text{L}}$	Reactive power on line bb' from bus b , in operating point t
θ_{bt}	Voltage phase angle at bus b , in operating point t
$d_{r,t}$	Target minus actual power supplied to region r at operating point t
$s_{r,t}$	Shortfall from load allocation in region r at operating point t
s_r^{L}	Long-term shortfall from load allocation in region r

Therefore the Short-term Load Allocation Problem is to:

$$\min W \sum_{r,t} s_{r,t} - \sum_{r,t} p_{r,t}^R \quad (1)$$

subject to

$$\sum_{g \in \mathcal{G}_b} p_{g,t}^G = \sum_{d \in \mathcal{D}_b} p_{d,t}^D + \sum_{b' \in \mathcal{B}_b} P_{bb',t}^L, \quad (2)$$

$$\sum_{g \in \mathcal{G}_b} q_{g,t}^G = \sum_{d \in \mathcal{D}_b} q_{d,t}^D + \sum_{b' \in \mathcal{B}_b} Q_{bb',t}^L, \quad (3)$$

$$p_{d,t}^D = (1 - \alpha_{d,t}) P_{d,t}^D, \quad (4)$$

$$q_{d,t}^D = (1 - \alpha_{d,t}) Q_{d,t}^D, \quad (5)$$

$$0 \leq \alpha_{d,t} \leq 1, \quad (6)$$

$$p_{r,t}^R = \sum_{d \in \mathcal{D}_r} p_{d,t}^D, \quad (7)$$

$$P_{bb',t}^L = v_{b,t}^2 g_{bb} + v_b v_{b'} (g_{bb'} \cos(\theta_{b,t} - \theta_{b',t}) + b_{bb'} \sin(\theta_{b,t} - \theta_{b',t})), \quad (8)$$

$$Q_{bb',t}^L = -v_{b,t}^2 b_{bb} + v_b v_{b'} (g_{bb'} \sin(\theta_{b,t} - \theta_{b',t}) - b_{bb'} \cos(\theta_{b,t} - \theta_{b',t})), \quad (9)$$

$$\theta_{b_0} = 0, \quad (10)$$

$$V_b^{\text{LB}} \leq v_{b,t} \leq V_b^{\text{UB}}, \quad (11)$$

$$P_g^{\text{LB}} \leq p_{g,t} \leq P_g^{\text{UB}}, \quad (12)$$

$$Q_g^{\text{LB}} \leq q_{g,t} \leq Q_g^{\text{UB}}, \quad (13)$$

$$P_{bb',t}^L{}^2 + Q_{bb',t}^L{}^2 \leq (S_{bb'}^{\text{max}})^2, \quad (14)$$

$$d_{r,t} = P_r \sum_{r'} p_{r',t}^R - p_{r,t}^R \quad (15)$$

$$\left. \begin{array}{l} s_{r,t} \geq 0 \\ s_{r,t} \geq d_{r,t} \end{array} \right\} \quad (16)$$

The Long-term Load Allocation Problem is to:

$$\min W \sum_r s_r^L - \sum_{r,t} p_{r,t}^R \quad (17)$$

subject to constraints (2) to (15) and also subject to

$$\left. \begin{array}{l} s_r^L \geq 0 \\ s_r^L \geq \sum_t d_{r,t} \end{array} \right\} \quad (18)$$

Equations (2)–(3) are Kirchhoff’s Current Law (KCL), enforcing real and reactive power balance, (4)–(6) define the power consumption in terms of the proportion $\alpha_{d,t}$ of real and reactive load shed at each demand bus d and operating point t , (8)–(9) are Kirchhoff’s Voltage Law (KVL), (10) removes the degeneracy in the bus voltage angles by fixing it to zero at the arbitrary reference bus, (11)–(13) are constraints on voltage and power generation and (14) is the line flow constraints.

The line conductance $g_{bb'}$ and susceptance $b_{bb'}$ are defined by

$$g_{bb'} = \frac{r_{bb'}}{r_{bb'}^2 + x_{bb'}^2}, \quad b_{bb'} = \frac{-x_{bb'}}{r_{bb'}^2 + x_{bb'}^2},$$

where $r_{bb'}$, $x_{bb'}$ are the line resistance and reactance, and parameters $G_{bb'}$ and $B_{bb'}$ are defined by

$$g_{bb'} = -\tau_{bb'}G_{bb'} = -\tau_{bb'}G_{b'b} = G_{b'b'} = \tau_{bb'}^2G_{bb}, \quad (19)$$

$$b_{bb'} + 0.5b_{bb'}^C = B_{b'b'} = \tau_{bb'}^2B_{bb}, \quad (20)$$

$$-b_{bb'} = \tau_{bb'}B_{b'b} = \tau_{bb'}B_{bb'}, \quad (21)$$

where $b_{bb'}^C$ is the line charging susceptance and $\tau_{bb'} = 1$ except in transformer ‘lines’, where it is the tap ratio and (as in the MATPOWER [20] convention) the ideal transformer is at the b end of the line.

3.2 Solution Method for Single Operating Points

In the simplest case where we consider just a single operating point of generation availability the shortfall as defined in the Short-term and Long-term problems are equivalent. We can obtain the Pareto frontier of the two objectives by optimising just with respect to the power supply objective and imposing a constraint on the total shortfall (This is the ϵ -constraint method described in [21]). This allows us to guarantee a good spread of points on the Pareto frontier and obtain points on concave parts of the Pareto frontier if they exist. In this approach we set W to a very small positive value so that the solver will prefer solutions with a smaller level of shortfall, all other things being equal.

3.3 Short-term Load Allocation Problem Solution Method

The Short-term Problem is separable by operating points. Each operating point defines an OPF which is independent from the others. We can see this by reformulating (1) as:

$$\sum_t \min W \sum_r s_{r,t} - \sum_r p_{r,t}^R \quad (22)$$

and noting that none of the constraints include more than one value of t .

We solve each problem using the Interior Point Method implemented in IPOPT [22]. The AC OPF is a non-linear, non-convex problem so there is no theoretical guarantee that the solver will converge to the globally optimal solution. In addition to converging to a locally optimal solution, the solver could converge to a stationary but non-optimal point, or converge to an infeasible point. However, methods that guarantee global optimality are not yet a practical approach for the size and structure of problems we consider (a meshed network of 637 buses in our case study). In practical experience, local optima of the AC OPF are relatively

rare [19]. Nevertheless we address the risk of finding a local optima by solving each cases from different initial points. For the first run we set $v_b = 1$ and $\delta_b = 0$ for all buses b , and $p_{g,t}$ and $q_{g,t}$ at the midpoint of their upper and lower bounds for each generator g . For subsequent runs the values of these decision variables were randomised. Using this method we have not been able to identify any local optima in the problem we consider in the case study.

3.4 Long-term Load Allocation Problem Solution Method

The Long-term Load Allocation Problem is not seperable by operating points because of the complicating constraint (18). The problem becomes intractable for large numbers of operating points. We therefore use an approach based on Lagrangian relaxation to decompose the problem. Because of the non-convexity of the original problem this approach is to some extent heuristic. However, as we show in Section 4, we can solve problems to a very small duality gap using this method. Lagrangian relaxation is applied to (18) (retaining the positivity constraint $s_r^L \geq 0$) and the objective (17) to get the Lagrangian function (with weighting parameter W determining the balance between the two primal objective terms):

$$\begin{aligned} L(\boldsymbol{\mu}, W) = \min W \sum_r s_r^L - \sum_{r,t} p_{r,t}^R - \sum_r \mu_r (s_r^L - d_{r,t}) = \\ \min W \sum_r s_r^L - \sum_{r,t} p_{r,t}^R - \sum_r \mu_r (s_r^L - P_r \sum_{r',t} p_{r',t}^R + \sum_t p_{r,t}^R) \end{aligned} \quad (23)$$

subject to constraints (2) to (15) and $s_r^L \geq 0$.

By the weak duality theorem this gives a lower bound on the primal problem for any choice of $\boldsymbol{\mu} \geq 0$. Furthermore this can be rearranged to:

$$L(\boldsymbol{\mu}, W) = \sum_r \min s_r^L (W - \mu_r) + \sum_t L_t(\boldsymbol{\mu}, W) \quad (24)$$

where,

$$L_t(\boldsymbol{\mu}, W) = \min \sum_r p_{r,t}^R (-1 + \sum_{r'} (\mu_{r'} P_{r'}) - \mu_r) \quad (25)$$

Therefore (23) is separable by operating points because the none of the constraints mention more than one value of t

For a given $\boldsymbol{\mu}$ we can solve $L(\boldsymbol{\mu}, W)$ by solving $|\mathcal{T}|$ independent OPF problems $L_t(\boldsymbol{\mu}, W)$, where \mathcal{T} is the set of operating points, and calculating the optimal $s_r^L \geq 0$ to minimise

$$\sum_r \min s_r^L (W - \mu_r)$$

The best lower bound will be given by $\max_{\boldsymbol{\mu}} L(\boldsymbol{\mu}, W)$, from which we can observe that the optimal $\boldsymbol{\mu} \in [0, W]$, otherwise $L(\boldsymbol{\mu}, W)$ is unbounded below.

To find the values for $\boldsymbol{\mu}$ which maximise $L(\boldsymbol{\mu}, W)$ we can build a cutting plane approximation to $L(\boldsymbol{\mu}, W)$. From the solution of $L(\boldsymbol{\mu}, W)$ for any particular $\boldsymbol{\mu}$ we can compute the gradient $\nabla_{\boldsymbol{\mu}} L(\boldsymbol{\mu}, w)$, from $\frac{\partial}{\partial \mu_r} L(\boldsymbol{\mu}, w) = \hat{s}_r - P_r \sum_{r'} \hat{p}_{r',t}^R + \sum_t \hat{p}_{r,t}^R$ which gives us the cutting plane $f \leq L(\boldsymbol{\mu}, W) + (\hat{\boldsymbol{\mu}} - \boldsymbol{\mu}) \nabla_{\boldsymbol{\mu}} L(\boldsymbol{\mu}, W)$. For a set of

cutting planes \mathbb{P} we solve the master problem $\max_{f \in \mathbb{R}} f$ subject to \mathbb{P} and $0 \leq \hat{\mu}_r \leq W$ to obtain an updated $\hat{\mu}$. In practice we find that a dual stabilisation method is needed for the dual problem to converge in reasonable time. We use a Proximal Bundle Method (see [23] for example).

4 CASE STUDY: NIGERIA

Nigeria exhibits the features that motivate our modeling approach: demand exceeds the power system's ability to supply power and the Nigerian regulator has adopted an approach to manage the regional rationing of power based on proportional Load Allocation targets for each distribution company.

In this case study we analyse how enforcing this regulation over different time scales and with varying levels of strictness affects the total amount of power that can be delivered. Requiring the regulation to be satisfied over a short time horizon during which the operating conditions of the power system are expected to remain constant corresponds to the Short-term Load Allocation Problem. Requiring the regulation to be satisfied on average, over the long term corresponds to the Long-term Load Allocation Problem. In order to explore the effect of enforcing the policy with varying levels of strictness we obtain the Pareto frontier of the two objectives: minimising the total load shed and minimising a measure of the deviation from the proportional targets in all regions that have a shortfall. We first consider each of the operating points individually and obtain the Pareto frontier of power supply and shortfall in each operating point. We then obtain Pareto frontier of both the Long-term and Short-term Load Allocation problems for this set of operating points.

4.1 Background on the Nigerian Power System

As a result of the recent privatisation and restructuring of the Nigerian power system there are 11 private utility companies with regional distribution and retail monopolies (DISCOs). The roles of the transmission service provider and system operator are combined in the government-owned Transmission Company of Nigeria (TCN). Generation is owned by privatised generation companies (GENCOs) but centrally dispatched by TCN and the DISCOs are required to pay the GENCOs for energy received according to generation tariffs set by the Nigerian Electricity Regulatory Commission (NERC).

The proportional load allocation targets set by NERC are shown in Table 2. These roughly correspond to the proportion of customers in each DISCO. The precise implementation of the load allocation policy has varied since privatisation, but the principle is that load supplied in each DISCO region over some time period should be a fixed proportion of the total load supplied.

The transmission system topography comprises a meshed grid in the south west of the country connected to the north and east by radial lines (see Figure 1). The nameplate capacity and mean available capacity of the generators are tabulated in Table 3.

TABLE 2: Target for percentage of total supply to be delivered to each Nigerian DISCO

DISCO	Load Allocation
Abuja	11.5%
Benin	9.0%
Eko	11.0%
Enugu	9.0%
Ibadan	13.0%
Ikeja	15.0%
Jos	5.5%
Kaduna	8.0%
Kano	8.0%
Port Harcourt	6.5%
Yola	3.5%

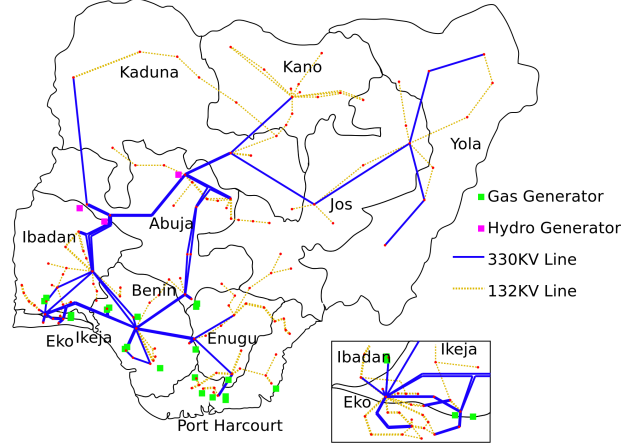


Fig. 1: Map of Nigeria showing distribution company areas, high voltage transmission lines and location of generators

4.2 Nigerian Network Model

We consider a 637 bus single phase equivalent circuit model of the Nigerian transmission system¹. The model contains data on the generation assets and the 330KV and 132KV transmission system. 111 buses are at the 330KV level, 176 at the 132KV level and 190 at the 33KV level. The remainder are low voltage load buses or generator buses. All the branches connecting to buses at a lower voltage than 132KV are transformers, not transmission lines. Loads are aggregated at grid supply points where the voltage is stepped down to distribution level. The model is intended to represent the state of the transmission network at the end of 2014. There has

1. This model was provided and validated by EMRC <http://energy-mrc.co.uk/>

TABLE 3: On-Grid generation capacity

Generator Name	Nameplate Capacity (MW)	Mean Installed Available Capacity (MW)
Gas Fired Generators		
AES	270	98
Afam IV-V	600	34
Afam VI	650	450
Delta	972	328
Egbin	1320	733
Geregu	414	145
Geregu	434	170
NIPP		
Ibom Power	190	51
Ihovbor	450	168
Okpai	450	398
Olorunsogo	335	140
Olorunsogo	675	169
NIPP		
Omoku	250	0
Omotosho	335	126
Omotosho	450	198
NIPP		
Rivers	150	94
Sapele	1020	70
Sapele NIPP	450	137
Hydro Generators		
Jebba	570	321
Kainji	760	142
Shiroro	600	309
On-Grid	11345	4281
Total		

been minimal transmission investment since privatisation, so by varying the generator availability the model can be taken to represent the state of the Nigerian power system at any point from November 2013 to the present.

4.3 Comparison of the AC and DC Models

We considered carrying out our analysis with the AC and DC versions of the models. Here we present the results that motivate our decision to use the AC model. Under both the AC and DC specifications of the problem we maximise supplied load without considering load allocation targets and then maximise supplied load with the load allocation targets strictly enforced. We repeat this for 26 generation scenarios in from 3 to

5.5 GW MW scaling the nameplate real power generation capacity of each generator by the same factor so that the spatial distribution of generation is the same. Figure 2 shows the total supplied load plotted against the available generation capacity in these 4 cases. A feasible solution could not be found for the generation scenarios with 3 and 3.1 GW of available capacity using the AC model with load allocation targets.

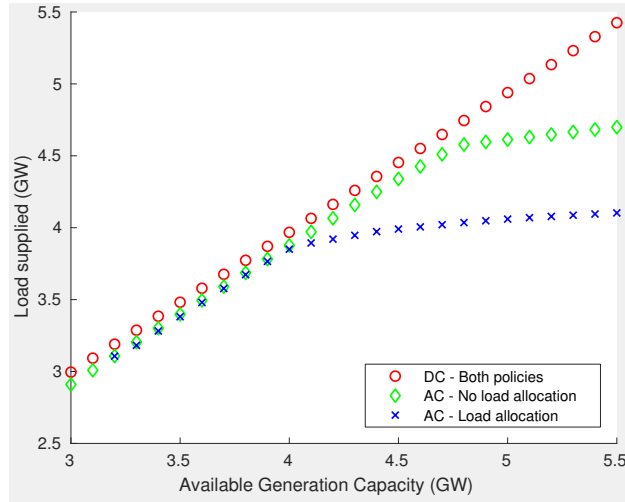


Fig. 2: Variation in supplied load with changes in generation availability under AC and DC models

More load is supplied in the solution to the DC model than is possible with the AC model, particularly for higher generation scenarios. Furthermore, in the DC model the load allocation policy has no effect on delivered load. We can see from the large difference in the results for the AC and DC models that accurate modeling of losses and reactive power flows is important to get accurate results for the Nigerian system. These results suggest that there is a shortage of reactive power in the network and that installation of reactive compensation could help to alleviate some of the transmission constraints.

On the basis of these results we carry out our analysis with the AC model. The DC approach could be modified by adding some approximation of the losses (e.g. a constant loss model) and using a slightly lower line flow limit to account for reactive power flows. However, it is unlikely that any approximation of losses and reactive power flows would give accurate results in the wide range of different operating conditions of the network that we explore.

4.4 Approximating the Variation in Network Operating Conditions

Our problem formulation is general enough that the set of operating points can represent any variation in the operating conditions of the network including generation availability, demand and transmission system topology. In the Nigerian case it is the variation in the level of available generation that is the major cause of supply variability. Variation in the network topology due to faults is of a lower order of significance. Demand is a source of uncertainty but this does not matter because we find that our results are not sensitive to the level of load within the range of all realistic load estimates (see results in the Appendix). Demand is sufficiently high

that transmission and generation constraints always prevent the load being fully supplied in any region. For these reasons we only consider variation of generation in this study.

We have obtained one set of measurements of actual connected load. In the absence of more data we assume that the measurements are representative of the power factor at grid supply points and the relative scale of the loads within distribution company regions. The measurements do not include the shed load, however, since load shedding is endogenous to our model, the level of power demand in the model must include the shed load as well. Therefore we scale the measurements of connected load up to a total of 10 GW maintaining the same power factor and ensuring that the total load in each DISCO is proportional to its load allocation target. This demand level was considered to be plausible by local experts and as we argued in the previous paragraph, the precise level is not important.

The level and distribution of generation available is exogenous to our model and it is highly variable. Figure 3 shows total available capacity over 1 year from May 2014. We have obtained a data set that gives the declared available capacity for individual generators on each of these days. We selected a set of 21 days uniformly spaced over the year starting May 2014 and create an operating point to match generation availability on that day by scaling the maximum generation output of each generators to match its availability on that day. This implicitly approximates the probability distribution of the level and spatial distribution of available generation. 21 operating points were chosen to balance the goals of minimizing computational complexity while approximating the space of past variation reasonably well.

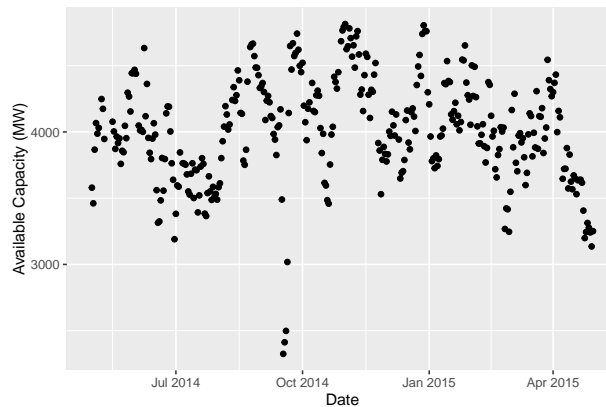


Fig. 3: Time series of available generation capacity

4.5 Results

Using the method described in Section 3.2, we have obtained the Pareto frontiers of the AC Load Allocation problem for all of the 21 selected operating points. An illustrative selection of these are shown in Figure 4. The horizontal axis in this chart measures the total deviation from the load allocation targets, while vertical axis measures the total load supplied. The different curves show Pareto frontiers between the two objectives for different generation operating points.

If the generation availability level is very low, the Pareto frontier is nearly flat, which means that shortfall can be eliminated with little cost in terms of total power supply. In these operating points power can be allocated to where policy makers have decided it should be without causing network congestion. The small decrease in overall power supplied is caused by slightly higher transmission losses. However, for the operating points with more available generation it is not possible to supply enough power to some regions, so rigorously adhering to the load allocation targets severely conflicts with maximising supply. This can be seen from the steep upward sloping curved region at the left hand end of these curves which indicated that gains in terms of reduced shortfall come at the cost of a significantly reduced overall power supply.

The particularly steep curves at very high generation availability levels illustrates a potential downside of a proportional policy of the kind described in this paper: the potential to create perverse incentives for the System Operator. If a region is in shortfall relative to its proportional target, but there is a hard transmission constraint preventing further power being supplied here, then the only way for that region's proportional target to be achieved would be to reduce the power supplied to other regions. In the Nigerian case we find that this happens in some individual operating points with high generation availability (over 4.5 GW) and a large weight on the objective of minimising shortfall. Available generation is very rarely this high in Nigeria, but if availability improves without expansion of transmission capacity, proportional regulation of this kind may cause problems.

We next compare the Pareto frontier for the Short-term and Long-term AC Load Allocation problems using the methods described in Sections 3.3 and 3.4. We have obtained 20 solutions to the Short-term Load Allocation Problem. The shortfall averaged over all operating points ranges from 2 MW to 380 MW. Reducing the average shortfall to 2 MW comes at the cost of 5% additional load shedding.

However, average shortfall as defined in the Long-term Load Allocation problem can be reduced to a similar level with much less load shedding. In our solutions to the Long-term problem the lowest shortfall obtained is 5 MW, which comes at the cost of only 2% of additional load shedding (see Table 4). Although our solution method for the Long-term Load Allocation problem can in theory leave a duality gap, we can see from the duality gaps reported in Table 4 that these points are very close to optimal. As a percentage of the objective function value the largest gap is 0.05% and most are much smaller than this.

In Figure 5 we plot all of the solutions obtained to the Short-term and Long-term Load Allocation Problems in terms of the average shortfall in each operating point. In this figure we approximate the Pareto Frontier by interpolating between these points.

For both problems there is a flat region on the right of the Pareto frontier indicating that these levels of shortfall can be reduced at little cost in terms of total power supply. However, in both cases there is also a steep downward sloping curved region at the left hand end, indicating that it is not possible to adhere to the load allocation targets without reducing the supply to regions where more could be supplied.

In the Short-term Problem the transition between the flat and sloped regions of the Pareto frontier occurs when the average shortfall has been reduced to 140 MW. This is a higher level than for the Long-term Problem where the transition occurs around 50 MW. Further reducing shortfall below these levels comes at an increasing cost in terms of load shedding. This implies that enforcing load allocation targets over a longer

time horizon is substantially more efficient, as may be expected. However, this also means that some regions will be undersupplied for a longer period of time, which may not be acceptable.

TABLE 4: Solutions to the Long-term Load Allocation Problem

W	0.001	0.01	0.05	0.1	0.3	0.5
Supply (GW)	4.040	4.039	4.036	4.030	4.020	4.012
Supply (% of maximum)	100%	100%	99.9%	99.7%	99.5%	99.3%
Shortfall (GW)	0.431	.334	0.194	0.104	0.048	0.026
Duality Gap (%)	0.00%	0.00%	0.00%	0.01%	0.03%	0.02%
W	0.8	1	2	4	6	6.5
Supply (GW)	4.009	4.008	4.003	3.993	3.969	3.958
% of maximum	99.2 %	99.2%	99.1%	98.8%	98.3%	98.0%
Shortfall (GW)	0.022	0.019	0.015	0.012	0.007	0.005
Duality Gap (%)	0.05%	0.01%	0.01%	0.01%	0.01%	0.01%

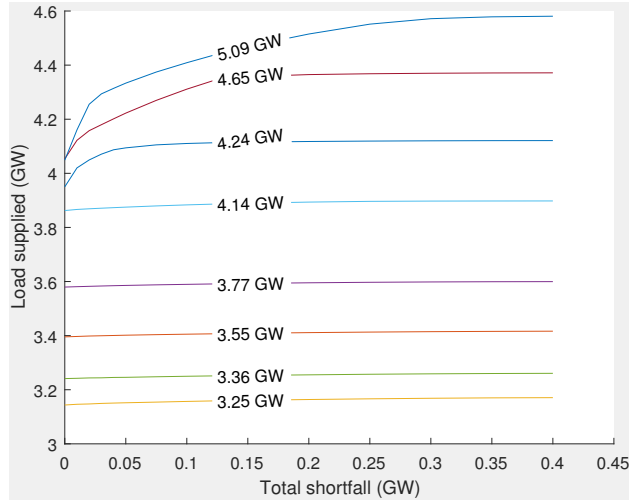


Fig. 4: Pareto frontiers for an illustrative selection of individual operating points labelled by total available generation

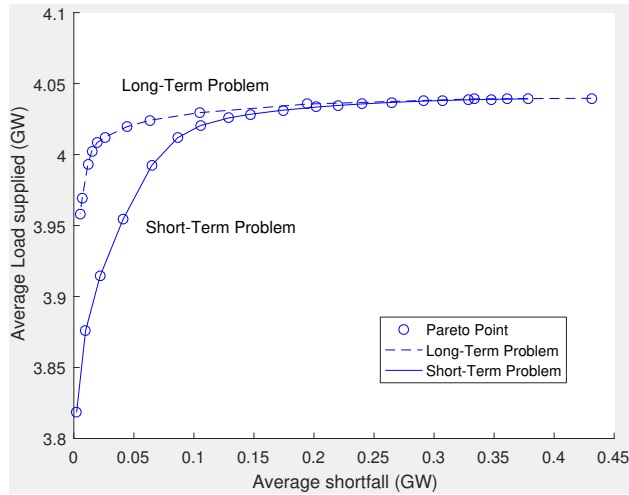


Fig. 5: Pareto optimal points of the Short-term and Long-term Load Allocation Problems

5 CONCLUSION AND POLICY IMPLICATIONS

In a power system in a long-term power shortage, some policy must be adopted that describes how to ration available power. If the System Operator is only incentivised to maximise delivered load, say by payments for transmission services, then they are unlikely to distribute power equally to different parts of the system, which may be seen as unfair or otherwise politically unacceptable. Countries with such power systems, such as Nigeria, therefore specify load allocation targets or similar metrics for each region. System Operators are then penalised for failing to meet these targets, incentivising them to choose a more equal distribution of the available power.

However, as we have argued in this paper, this can come at a substantial cost in terms of additional load shedding. Depending on the design of the policy the System Operator may be incentivised to minimise shortfall at every operating point or attempt to balance shortfall or surplus over the long term. It may also be allowed to deviate from load allocation targets to a certain extent. Policy makers need to know what the efficiency penalties are in each of these cases to design an optimal load allocation policy.

We have developed methods which can be used to quantify the trade-off between different objectives and implement a chosen policy, and applied this to the Nigerian power system. Our case study shows that the cost of the Nigerian load allocation targets is substantial, as it reduces the total amount of power supplied by up to 5%. This does not imply that the policy is inefficient, as long as a more equal distribution of the available power has substantial benefits, but without this quantification a trade-off between fairness and efficiency cannot be made.

It could be argued that instead of finding a balance between proportionality and maximum load, the System Operator should attempt to maximise a utility function of power supply. The utility of power in each region of the network is likely to have diminishing returns, so maximising the sum of the utility functions in all regions is likely to enforce some level of proportionality between regions in any case.

Alternatively the System Operator could have fixed targets for the minimum level of power to be supplied in each region, rather than proportional targets such that the target moves, in absolute terms, with the total level of power supply. Once these targets have been met, surplus power could be distributed according to some other rule. This could solve the problem where proportional regulation encourages the System Operator meet the load allocation target in a given region by shedding load elsewhere even though it is not possible to supply more load in the region in question.

Nevertheless, in both these cases there remains a similar difficulty of trying to achieve these objectives subject to different network conditions as the level and spatial distribution of available generation varies. Furthermore, both of these objectives have Short-term and Long-term versions analogous to the Short-term/Long-term Load Allocation Problems described above. Therefore the methods we describe in this paper could be adapted to these alternative problems.

In the course of our investigation we found that investment in reactive compensation could make it feasible to achieve the Nigerian load allocation targets with less load shedding. However, it should be noted that although reactive support is cheap relative to generation or transmission lines, developing world power systems often find it difficult to meet even this level of investment. It is therefore important to consider how to most efficiently operate and regulate the existing network. Although alleviating these problems by investment in reactive support is beyond the scope of this paper our method could be used to evaluate different plans for investment in reactive support. An interesting subject of further research would be to optimise the siting of capacitor banks.

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APPENDIX

The level of unsuppressed load (demand without load shedding) is the parameter of the model that has the greatest range of uncertainty. Therefore sensitivity analysis of these results was carried out to quantify the sensitivity of our model to potential inaccuracy of this estimate. We might expect that in spite of the transmission constraints, some regions of the network can absorb more power. If so the total load that can be supplied may also increase if the loads in these regions are increased.

The optimal power flow model was solved for 21 individual operating points with overall load level ranging from 5 GW to 15 GW increments of .5 GW (Note, these are not the same 21 operating points used in the case study). In the first instance the optimisation was carried out with an objective to minimise load shedding with no load allocation targets. In the second case we require strict adherence to the proportional load allocation targets. The individual loads at each bus were scaled uniformly. The analysis was carried out for 6 levels of generation availability: 3, 3.5, 4, 4.5, 5 and 5.5 GW. In each case we scale the nameplate real and reactive power generation capacity of each generator by the same factor so that the spatial distribution of generation is the same.

The results of maximising total supply are shown in figure 6 where total load supplied is plotted against unsuppressed load for all 6 scenarios. A horizontal line for a given scenario means that the level of load supplied does not increase if the level of unsuppressed load is higher. The 5 and 5.5 GW generation availability operating points display the highest sensitivity to the load level. The 3, 3.5, 4 GW operating points display very little sensitivity to the level of load while the 4.5 GW scenario displays some sensitivity to the level of load between 5 and 7 GW.

The results when we minimise the shortfall from the proportional targets are shown in figure 7. Again the sensitivity to load level is largest in the lower range of unsuppressed load (5 to 10GW). (Note that with strict adherence to load allocation targets it was not possible to find any feasible solutions for the lowest generation scenario with 3 GW of available capacity.)

As shown in Figure 3 generation availability in the study period is very rarely in excess of 4.5 GW and never exceeds 5 GW. Therefore in the most realistic range of generation scenarios (3 to 4.5 GW) our results are not sensitive to the estimate of load as long as it is in excess of 7 GW. Based on discussions with consultants working in the sector we have verified that demand is at all times likely to be in excess of 7 GW. We conclude that our results would not be significantly affected by considering a range of different demand estimates or variation in demand across the day.

However the methods we have developed could easily accommodate a situation in which modeling variation in demand over the day is more important. For example one could replicate each generation operating point with three operating points with a peak, off-peak and medium demand estimate and weight these in the objective function according to the proportion of the day they represent.

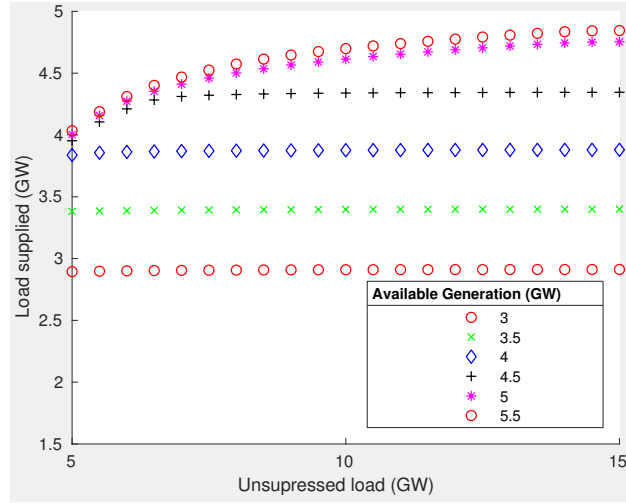


Fig. 6: Total load supplied as a function of unsuppressed load without load allocation targets

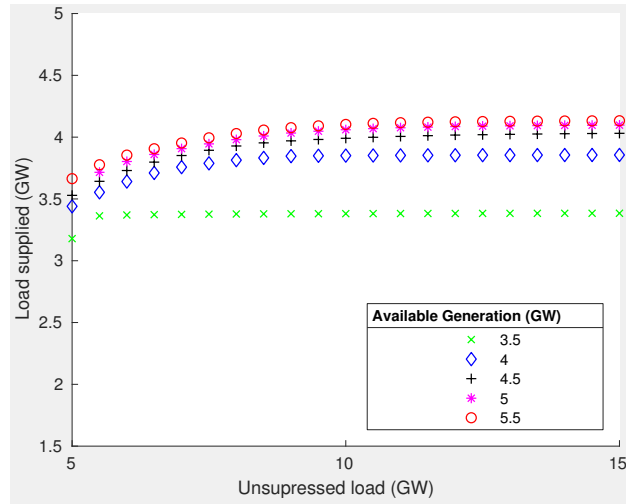


Fig. 7: Total load supplied as a function of unsuppressed load with load allocation targets strictly enforced

REFERENCES

- [1] Z. Gatugel Usman, S. Abbasoglu, N. Tekbiyik Ersoy, and M. Fahrioglu, "Transforming the Nigerian power sector for sustainable development," *Energy Policy*, vol. 87, pp. 429–437, dec 2015.
- [2] I. N. Kessides, "Chaos in power: Pakistan's electricity crisis," *Energy Policy*, vol. 55, pp. 271–285, apr 2013.
- [3] Econ, "The future is black," jan 2012. [Online]. Available: <http://www.economist.com/node/21543138> [Accessed: 2018-02-03]
- [4] L. Rohter, "Electricity Rationing in Brazil Inflames Regional Animosities," nov 2001. [Online]. Available: <http://www.nytimes.com/2001/11/04/world/electricity-rationing-in-brazil-inflames-regional-animosities.html> [Accessed: 2018-02-03]
- [5] P. Burkhardt and F. Wild, "South Africa Blackouts Ended by Economic Slowdown, Not Eskom," 2016. [Online]. Available: <http://www.bloomberg.com/news/articles/2016-03-13/south-african-blackouts-ended-by-economic-slowdown-not-eskom> [Accessed: 2018-02-03]

- [6] A. Mukaro, "Zimbabwe: Zesa to Introduce Load-Shedding," feb 2003. [Online]. Available: <http://allafrica.com/stories/200302210432.html> [Accessed: 2018-02-03]
- [7] BBC, "Serbs face blackout misery," 2000. [Online]. Available: <http://news.bbc.co.uk/1/hi/world/europe/1088352.stm> [Accessed: 2018-02-03]
- [8] T. B. Andersen and C.-J. Dalgaard, "Power outages and economic growth in Africa," *Energy Economics*, vol. 38, pp. 19–23, jul 2013.
- [9] M. Marchand, "Pricing power supplied on an interruptible basis," *European Economic Review*, vol. 5, no. 3, pp. 263–274, jan 1974.
- [10] S. S. Oren and J. A. Doucet, "Interruption insurance for generation and distribution of electric power," *Journal of Regulatory Economics*, vol. 2, no. 1, pp. 5–19, mar 1990.
- [11] J. A. Doucet, K. Jo Min, M. Roland, and T. Strauss, "Electricity rationing through a two-stage mechanism," *Energy Economics*, vol. 18, no. 3, pp. 247–263, 1996.
- [12] J.-T. Bernard and M. Roland, "Load management programs, cross-subsidies and transaction costs: the case of self-rationing," *Resource and Energy Economics*, vol. 22, no. 2, pp. 161–188, 2000.
- [13] H. Ahlborg, F. Boräng, S. C. Jagers, and P. Söderholm, "Provision of electricity to African households: The importance of democracy and institutional quality," *Energy Policy*, vol. 87, pp. 125–135, dec 2015.
- [14] T. S. P. Fernandes, J. R. Lenzi, and M. A. Mikilita, "Load Shedding Strategies Using Optimal Load Flow With Relaxation of Restrictions," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 712–718, may 2008.
- [15] D. Ding Xu and A. Girgis, "Optimal load shedding strategy in power systems with distributed generation," in *2001 IEEE Power Engineering Society Winter Meeting. Conference Proceedings (Cat. No.01CH37194)*, vol. 2. IEEE, 2001, pp. 788–793.
- [16] J. Zhu, "Optimal Load Shedding," in *OPTIMIZATION OF POWER SYSTEM OPERATION*. Hoboken, NJ, USA: John Wiley & Sons, Inc, jan 2015, pp. 437–482.
- [17] The World Bank, "Implementing power rationing in a sensible way : lessons learned and international best practices," 2005.
- [18] B. Tenenbaum and J. Besant-Jones, "The California power crisis : Lessons for developing countries," p. 1, 2001.
- [19] W. A. Bukhsh, A. Grothey, K. I. M. McKinnon, and P. A. Trodden, "Local Solutions of the Optimal Power Flow Problem," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4780–4788, nov 2013.
- [20] R. D. Zimmerman, C. E. Murillo-Sanchez, and R. J. Thomas, "MATPOWER: Steady-State Operations, Planning, and Analysis Tools for Power Systems Research and Education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, feb 2011.
- [21] Vira Chankong. and Y. Y. Haines, *Multiobjective decision making : theory and methodology*. Dover Publications, 2008.
- [22] A. Wächter and L. T. Biegler, "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," *Mathematical Programming*, vol. 106, no. 1, pp. 25–57, mar 2006.
- [23] J.-B. Hiriart-Urruty and C. Lemarechal, *Convex Analysis and Minimization Algorithms II : Advanced Theory and Bundle Methods*. Springer Berlin Heidelberg, 1993.